

An interplay between static potential and Reggeon trajectory for QCD string*

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Abstract

I consider two cases where QCD string is described by an effective theory of long strings: the static potential and meson scattering amplitudes in the Regge regime. I show how they can be solved in the mean-field approximation, justified by the large number of space-time dimensions, and argue that it turns out to be exact. I compare contributions from QCD string and perturbative QCD and discuss experimental consequences for the scattering amplitudes.

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I. INTRODUCTION

QCD string is formed at the distances larger than the confinement scale and is described by an effective string theory, which has a well-defined semiclassical expansion around a long-string ground state. The leading order, known as the Lüscher term [1], is universal and possesses a number of remarkable properties, owing to the fact that the transverse size of the long string is also large. The number of transverse degrees of freedom of the effective string resides in an overall coefficient of the Lüscher term, which is the only input parameter that equals $d-2$ for a bosonic string in d space-time dimensions. A long-standing idea is to extract physical consequences directly from this well-established picture of QCD string.

I consider in this Letter two cases where QCD string is outstretched, so that the effective string theory is applicable: a rectangular Wilson loop, determining the static potential, and meson scattering amplitudes in the Regge kinematical regime, determining the Reggeon trajectory. I emphasize similarities between these two cases and describe how they can be solved in the mean-field approximation, which is justified by large d , but turns out to be exact at any d . I take into account contributions to the scattering amplitudes from both QCD string and perturbative QCD and discuss some experimental consequences.

II. STATIC POTENTIAL

A linear static potential $V(R)$ between heavy quarks in QCD corresponds to the area-law behavior of a $R \times T$ rectangular Wilson loop

$$W(R \times T) = e^{-TV(R)} \quad (1)$$

for $T \gg R$ and results in quark confinement. This implies that the gluon field forms a flux tube or a string for R larger than the confinement scale.

In string theory a linear potential is associated with a classical string, while a quantization of an outstretched string with the Dirichlet boundary condition in d dimensions yields [2, 3]

$$V(R) = \frac{1}{2\pi\alpha'} \sqrt{R^2 - \pi^2 \frac{(d-2)}{6} \alpha'}, \quad (2)$$

where $1/2\pi\alpha'$ is the string tension. Equation (2) was first obtained [2] in the large- d limit, where only transverse degrees of freedom are essential, and then derived [3] by the Virasoro quantization of the string with the Dirichlet boundary condition. What was most important

with the formula in Eq. (2) was a recognition of the fact that it to be exact for any d order by order in $1/R$, since the Lorentz anomaly vanishes at large R so the quantization is consistent [4]. The root in Eq. (2) becomes imaginary for small distances, which is related to tachyonic instability, but there is no tachyon for long strings.

For a covariant quantization the Lorentz anomaly is translated into the Virasoro anomaly. It is now possible to describe QCD string in $d < 26$ by an effective string theory with a nonpolynomial action [5]

$$S_{\text{eff}} = \frac{1}{\pi\alpha'} \int d^2z \partial X \cdot \bar{\partial} X + \frac{(d-26)}{24\pi} \int d^2z \frac{\partial^2 X \cdot \bar{\partial}^2 X}{\partial X \cdot \bar{\partial} X} + \dots, \quad (3)$$

where the conformal anomaly is expressed (modulo total derivatives and the constraints) via an induced metric

$$e^{\varphi_{\text{ind}}} = 2 \partial X \cdot \bar{\partial} X, \quad (4)$$

which is not treated independently as distinct from the Polyakov formulation. It was shown [5, 6] by analyzing the effective string theory order by order in $1/R$ that the Virasoro anomaly vanishes and the spectrum reproduces that [3] of the Nambu–Goto string in d dimensions, which remarkably agrees with the recent lattice calculations [7].

III. MOMENTUM-SPACE DISK AMPLITUDE

The consideration of scattering amplitudes for QCD string is pretty much similar to the above analysis of the static potential. They are given by a momentum-space disk amplitude for (smeared) stepwise [8]

$$p^\mu(t) = \frac{1}{\pi} \sum_i p_i^\mu \arctan \frac{(t - t_i)}{\varepsilon_i} \rightarrow \frac{1}{2} \sum_i p_i^\mu \text{sign}(t - t_i), \quad (5)$$

where $-\infty < t < +\infty$ is a parametrization of the momentum-space loop $p^\mu(t)$ and ε_i will play the role of a regularization. The (smeared) discontinuities of the step function (5) correspond to momenta of colliding particles (all of them are considered as incoming). Such a momentum-space loop is closed as a consequence of the momentum conservation. For $d = 26$, when the action is quadratic, this momentum-space amplitude coincides with the usual coordinate-space disk amplitude for the polygonal loop

$$x^\mu(t) = 2\pi\alpha' p^\mu(t), \quad (6)$$

whose vertices x_i^μ obey $x_{i+1}^\mu - x_i^\mu = 2\pi\alpha' p_i^\mu$. It looks like the polygon with light-like edges in Ref. [9] for $\mathcal{N} = 4$ super Yang–Mills.

For a $2 \rightarrow 2$ process it is convenient to consider a u -channel kinematics, when Mandelstam's variables $s, t < 0$ and $u > 0$. Keeping in mind an application to high-energy scattering, we can set $p_i^2 = 0$ for $-s \gtrsim -t \gg p_i^2$. At the classical level, we consider a harmonic extension of the boundary function (5) into the upper half-plane (UHP) $z = x + iy$:

$$X^\mu(x, y) = 2\alpha' \sum_i p_i^\mu \arctan \frac{(x - s_i)}{y}, \quad (7)$$

where $s_i = s(t_i)$ for a certain reparametrizing function $s(t)$ obeying $ds/dt \geq 0$.

For the function (7) the quadratic part of the action (3) equals

$$\frac{1}{2\pi\alpha'} S_{\text{quad}} = \alpha' s \ln r + \alpha' t \ln(1 - r), \quad (8)$$

where

$$r = \frac{s_{43}s_{21}}{s_{42}s_{31}}, \quad s_{ij} = s_i - s_j \quad (9)$$

is the projective-invariant ratio. The minimal surface is obtained by minimizing (8) with respect to r , which gives

$$r_* = \frac{s}{s + t}. \quad (10)$$

This is nothing but the well-known saddle point of the Veneziano amplitude, that reproduces the Regge behavior

$$A \propto e^{-S_{\text{min}}/2\pi\alpha'} \xrightarrow{s \gg t} e^{\alpha' t \ln(s/t)}. \quad (11)$$

IV. SEMICLASSICAL REGGEON INTERCEPT

The Regge behavior (11) with a linear trajectory $\alpha(t) = \alpha' t$ of zero intercept is associated with a classical string. For a long string, quantum fluctuations can be taken into account in a semiclassical approximation resulting in the Lüscher term [1], which is well-known for a $R \times T$ rectangle with $T \gg R$:

$$W(C) \stackrel{\text{rectangle}}{\propto} e^{-RT/2\pi\alpha' + \pi(d-2)T/24R}. \quad (12)$$

We shall now demonstrate how this shifts the Reggeon intercept.

UHP can be mapped onto a rectangle by the Schwarz–Christoffel formula

$$\omega(z) = \sqrt{s_{42}s_{31}} \int_{s_2}^z \frac{dx}{\sqrt{(s_4 - x)(s_3 - x)(x - s_2)(x - s_1)}}, \quad (13)$$

where the normalization factor is introduced for the projective symmetry. The new variable ω takes values inside a $\omega_R \times \omega_T$ rectangle with

$$\omega_R = 2K \left(\sqrt{1-r} \right) \xrightarrow{r \rightarrow 1} \ln \frac{16}{1-r}, \quad \omega_T = 2K \left(\sqrt{r} \right) \xrightarrow{r \rightarrow 1} \pi, \quad (14)$$

where K is the complete elliptic integral of the first kind. This has the meaning of a worldsheet parametrization.

To calculate the Lüscher term, we decompose

$$X^\mu(\omega_1, \omega_2) = X_{\text{cl}}^\mu(\omega_1, \omega_2) + Y_{\text{q}}^\mu(\omega_1, \omega_2), \quad (15)$$

where X_{cl}^μ is harmonic with the boundary value (5), so Y_{q}^μ has the mode expansion

$$Y_{\text{q}}^\mu(\omega_1, \omega_2) = \sum_{m,n} \chi_{mn}^\mu \sin \frac{\pi m \omega_1}{\omega_R} \sin \frac{\pi n \omega_2}{\omega_T}. \quad (16)$$

Now the Lüscher term results from the determinant coming from the path integral over Y_{q}^μ .

It is clear that each set of modes results in the Lüscher term

$$\frac{\pi \omega_T}{24 \omega_R} = \frac{1}{24} \ln \frac{16s}{t} \quad (17)$$

for $\omega_T \gg \omega_R$. There are $(d-2)$ such sets, so their contribution to the intercept of the Regge trajectory is [10, 11]

$$\alpha(0) = \frac{d-2}{24}. \quad (18)$$

It worth commenting on the way how the (same) Lüscher term emerges for the UHP parametrization, when the path integral over Y_{q}^μ does *not* depend on the boundary contour $p^\mu(t)$. In the critical dimension $d = 26$ it comes [12] now from the path integral over reparametrizations (or the boundary metrics), while for $d \neq 26$ the classical part X_{cl}^μ in the decomposition (15) additionally contributes through the second term on the right-hand side of Eq. (3), rather than the quantum part Y_{q}^μ :¹

$$\alpha(0) = 1 + \frac{d-26}{24} = \frac{d-2}{24}, \quad (19)$$

reproducing Eq. (18). How this happens for plane contours is described in the original paper [1], where the determinant of the Laplace operator is expressed via the metric induced

¹ One may wonder why the second term in the action (3) does not contribute to the Lüscher term for a rectangle, as the first term does. This can be verified by evaluating the contribution of the second term to the determinant, coming from the quadratic fluctuations of Y_{q}^μ in Eq. (15), whose trace of the log involves $\zeta(-2) = 0$ contributed by the second term.

by the conformal mapping $w(z)$. For this reason the calculation of the Lüscher term for UHP is pretty much similar to that of Ref. [13] for the contribution of the Liouville field in the Polyakov formulation. This is because the Liouville field can be simply substituted to the given order of the semiclassical expansion by its value given by the induced metric (4).

V. MEAN-FIELD APPROXIMATION

As was shown by Alvarez [2], a path integral of the Nambu–Goto string for a $T \times R$ rectangle is calculable in the limit of the large d by a saddle-point technique. The saddle-point value of the (mean-field) action is²

$$\frac{1}{2\pi\alpha'} S_{\text{mf}*} = \frac{T}{2\pi\alpha'} \sqrt{R^2 - \pi^2 \frac{(d-2)}{6} \alpha'}, \quad (20)$$

reproducing Eq. (2). As is already mentioned, the Virasoro quantization of the Dirichlet string shows [3] this formula to be exact and the quantization is consistent [4] order by order in $1/R$. This apparently means that the mean field approximation, which is usually justified by large d , turns out to be *exact* for any $d > 2$.

Equation (20) can be easily rederived in the conformal gauge using the worldsheet parametrization, when the expansion goes around the mean-field configuration

$$X_{\text{cl}}^1 = \frac{\omega_1}{\omega_R} R, \quad X_{\text{cl}}^2 = \frac{\omega_2}{\omega_T} T. \quad (21)$$

Here, ω_R and ω_T change under reparametrizations and have to be considered as variational parameters. Only the ratio ω_T/ω_R will be essential in what follows.

The mean-field action then takes the form

$$\frac{1}{2\pi\alpha'} S_{\text{mf}} = \frac{1}{4\pi\alpha'} \left(R^2 \frac{\omega_T}{\omega_R} + T^2 \frac{\omega_R}{\omega_T} \right) - \frac{\pi(d-2)}{24} \frac{\omega_T}{\omega_R}, \quad (22)$$

where we have also accounted for the Lüscher term. There are no corrections to this formula as $R^2 \sim d \rightarrow \infty$. Minimizing Eq. (22) with respect to ω_T/ω_R , we find

$$\left(\frac{\omega_T}{\omega_R} \right)_* = \frac{T}{\sqrt{R^2 - \pi^2 \frac{(d-2)}{6} \alpha'}}. \quad (23)$$

The substitution into Eq. (22) now reproduces Eq. (20).

² We keep here and below $(d-2)$ for the physical number of transverse degrees of freedom.

The above mean-field calculation can be repeated for the UHP parametrization, which is commonly used for representing scattering amplitudes in string theory, by making use of the Schwarz–Christoffel mapping (13). The ratio ω_T/ω_R , given by the ratio of elliptic integrals in Eq. (14), is known as the Grötzsch modulus which is monotonic in r . For this reason the varying with respect to r is equivalent to the varying with respect to ω_T/ω_R and Eq. (20) is reproduced.

We are now in a position to consider the scattering amplitude, when the Mandelstam variables s and t play the role of T and R . We then have

$$\frac{1}{2\pi\alpha'} S_{\text{mf}} = \alpha' s \ln r + \alpha' t \ln(1-r) + \frac{(d-2)}{24} \ln(1-r), \quad (24)$$

where we have added to Eq. (8) the associated momentum-space Lüscher term [10, 11]. There are again no corrections to Eq. (24) as $R^2 \sim d \rightarrow \infty$.

Minimizing the right-hand side of Eq. (24) with respect to r , we find

$$r_* = \frac{s}{s+t+(d-2)/24\alpha'} \quad (25)$$

which results for $s \gg -t$ in the Regge behavior

$$A \propto e^{\alpha(t) \ln(s/t)} \quad (26)$$

with the linear trajectory

$$\alpha(t) = \frac{(d-2)}{24} + \alpha' t. \quad (27)$$

It is obtained for large d but is expected to be exact for any d as is already pointed out. The arguments in favor of this conjecture are:

- 1) it is true in the semiclassical approximation;
- 2) it reproduces the exact result in $d = 26$;
- 3) it agrees with the existence [14] of a massless bound state in $d = 2$ large- N QCD for massless quarks.

The quadratic fluctuations around this mean field are stable for $\alpha(t) < 0$, i.e. for

$$-\alpha' t > \frac{d-2}{24}. \quad (28)$$

VI. MEAN-FIELD APPROXIMATION (CONTINUED)

It is instructive to compare the form (26) of the momentum-space disk amplitude with Eq. (1) for the $R \times T$ rectangular Wilson loop. Then $\ln(s/t)$ is an analog of T and $-\alpha(t)$

is an analog of $V(R)$. An important difference is however that Eq. (2) involves the square root, while $\alpha(t)$ is linear in t .

As is well known, the potential (2) is ill-defined for $R < R_c = \pi\sqrt{(d-2)\alpha'}/6$ because of the tachyonic singularity. What happens at $R = R_c$ can be understood by calculating the ratio of the area of the dominant surface at the saddle point to the minimal area spanned by the rectangle [2]

$$\left\langle \sqrt{\det g} \right\rangle_{\text{mf}} = \frac{2\pi(\alpha')^2}{S_{\text{min}}} \frac{d}{d\alpha'} \ln W = \frac{1-\lambda}{\sqrt{1-2\lambda}}, \quad \lambda = \pi^2 \frac{(d-2)\alpha'}{12R^2}. \quad (29)$$

It diverges when $R \rightarrow R_c$ from above, which means that typical surface becomes very large and the mean-field approximation ceases to be applicable.

The situation is different for the scattering amplitude, when the ratio of the area of the dominant surface at the saddle point to the minimal area spanned by the polygon is

$$\left\langle \sqrt{\det g} \right\rangle_{\text{mf}} = \frac{\alpha(t)}{\alpha't} = 1 + \frac{(d-2)}{24\alpha't}. \quad (30)$$

It vanishes rather than diverges when $-\alpha't$ is approaching the value on the right-hand side of Eq. (28) from above, so the linear Reggeon trajectory (27) can be analytically continued to smaller values of $-\alpha't$.

VII. APPLICATION TO QCD

QCD string is stretched between quarks, when they are moved apart, and makes sense of an effective string theory. Meson scattering amplitudes in large- N QCD are expressed through a sum over paths of the Wilson loop, which reduces [15] to the above momentum-space disk amplitude³, when quarks are massless and/or the number of colliding particles is very large. Therefore, the above Reggeon trajectory is of physical relevance for QCD.

The linear trajectory (27) cannot extend to very large values of $-\alpha't$, where perturbative QCD (pQCD) is applicable. It was shown [16] that the reggeization of the $\bar{q}q$ trajectory in pQCD is due to double logarithms coming (in the axial gauge) from ladder diagrams and resulting in the $2 \rightarrow 2$ amplitude

$$A = \frac{2I_1(\omega \ln(s/\mu^2))}{\omega \ln(s/\mu^2)} - 1 \propto e^{\omega \ln(s/\mu^2)} \quad \omega = \sqrt{\frac{g^2(t)C_F}{2\pi^2}} \approx .5, \quad (31)$$

³ The string disk amplitude is associated as usual with planar QCD diagrams and the $\bar{q}q$ Reggeon trajectory.

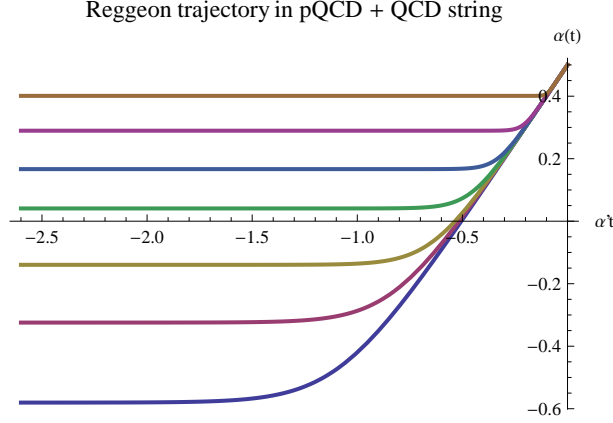


FIG. 1: Effective ρ Regge trajectory for $R = 23$.

where I_1 is the Bessel function and $\mu \sim 1$ GeV is an infrared cutoff in pQCD. Strictly speaking, the running of $g^2(t)$ with t is beyond the accuracy of the calculation, but it is expected from asymptotic freedom and provides the asymptote $\omega \rightarrow 0$ as $t \rightarrow -\infty$, which follows from the quark counting rule.

In the sum-over-path representation of meson scattering amplitudes in QCD the separation of pQCD and QCD string can be performed by splitting the integral over the proper time into two domains: $\mathcal{T} \lesssim 1/\mu$ and $\mathcal{T} \gtrsim 1/\mu$. The domain $\mathcal{T} \lesssim 1/\mu$ is associated with small paths and, correspondingly, pQCD, while domain $\mathcal{T} \gtrsim 1/\mu$ is associated with large paths and, correspondingly, QCD string. The total amplitude is the sum of the contributions from both domains:

$$A = \frac{2I_1(\omega \ln(\alpha' s))}{\omega \ln(\alpha' s)} - 1 + R(\alpha' s)^{\alpha(0) + \alpha' t}, \quad (32)$$

where R is a constant and we have set $\mu^2 = 1/\alpha'$.

For infinite s the first term on the right-hand side of Eq. (32) should dominate, so the Reggeon trajectory, which is linear for $t > 0$ with the slope $\alpha' \approx 1$ GeV from the measured spectrum of mesons, has to be (almost) constant $\alpha(t) \approx 0.5$ for $t < 0$, as it follows from the asymptotic behavior of I_1 . Such a behavior would not agree with the experimental data for the π^0 production in inclusive or exclusive processes at existing energies, as was pointed out, respectively, in Refs. [17, 18]. At finite s (but $s \gg -t$) both terms on the right-hand side of Eq. (32) are important, while their relative strength depends on the value of R .

In Fig. 1 we plot the effective Reggeon trajectory

$$\alpha_{\text{eff}}(t) = \frac{\ln(A/R)}{\ln(\alpha' s)} \quad (33)$$

for various $\alpha's$: $\alpha's = 400, 10^3, 10^4, 10^6, 10^9, 10^{16}$, and 10^{40} from the bottom to the top. The value of $R = 23$ is taken to fit the data [19] for $s = 400 \text{ GeV}^2$, which are described by the lower line in the figure. We have also substituted $\alpha(0) = 0.5$ to agree with the experimental data both for the meson masses and for the scattering processes.⁴

It is seen from Fig. 1 that for finite s the linear Reggeon trajectory continues from $t > 0$ to $t < 0$ and then flattens. The smaller s the deeper is the linear part at $t < 0$. A prediction of this figure is that $\alpha_{\text{eff}}(t) \approx 0$ for $s = 1 \text{ TeV}^2$. For s larger than 10^3 TeV^2 it is rising with s very slowly.

VIII. DISCUSSION AND OUTLOOK

We have considered in this Letter the two cases, where QCD string is outstretched: the static potential and the Regge regime of meson scattering amplitudes. They can be described by an effective string theory, which is an effective description for the degrees of freedom of long strings. In this situation it does not matter what is the actual QCD string (if any) which makes sense for all distances. Two such candidates for QCD string are:

- Migdal's elfin string [20], whose worldsheet is populated by two-dimensional elementary fermions;
- holography⁵ based on the AdS/CFT correspondence in a confining background, where extra degrees of freedom are described by higher dimensions.

In both cases the extra degrees of freedom are needed to provide asymptotic freedom at small distances, but at the distances larger than the confinement scale the mean-field approach apparently works well, reproducing the same results as for the Nambu-Goto string. This issue will be addressed elsewhere.

The ways the Dirichlet disk amplitude (reproducing the static potential) and the scattering amplitude (reproducing the linear Reggeon trajectory) are considered are pretty much similar, although the latter refers, strictly speaking, to scattering of tachyons, when the reparametrization path integral decouples. In QCD we deal with off-shell scattering amplitudes, when the reparametrization path integral plays a crucial role in maintaining the

⁴ This value of the Reggeon intercept is expected to emerge due to spontaneous breaking of the chiral symmetry in QCD and supersedes the one in Eq. (18), though it is not yet shown how this happens.

⁵ See Ref. [21], where this approach is compared to the lattice calculations.

projective symmetry. The path integral over reparametrizations obeying $s(t_i) = s_i$ yields [8]

$$\frac{\int \mathcal{D}_{\text{diff}} s G(s_j, s_j)}{\int \mathcal{D}_{\text{diff}} s} = \frac{1}{\pi} \ln \frac{(s_{j+1} - s_{j-1})}{(s_{j+1} - s_j)(s_j - s_{j-1})\varepsilon} \quad (34)$$

for the (singular) Green function at coinciding arguments, which is associated with the Lovelace Reggeon vertex that results in consistent off-shell scattering amplitudes [22], reproducing the above results.

There is, however, a big difference in the square-root behavior of the static potential (2) and the linear Reggeon trajectory (27) of QCD string. The former is linked to the tachyon and the $d = 1$ barrier, while the latter is apparently associated with smooth surfaces.

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